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LETTER TO THE EDITOR

DISCUSSION ON: "PROPAGATION OF TORSIONAL SURFACES WAVES IN VISCOELASTIC MEDIUM"

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It is shown in the following that the conclusion obtained by Dey *et al.*[1] about the existence of torsional surfaces waves in a homogeneous viscoelastic medium is incorrect. In fact, they do not demonstrate their existence, but merely find the phase velocity and attenuation factor of the torsional **body** wave.

They use a Kelvin–Voigt viscoelastic constitutive equation, whose viscoelastic complex modulus is[2]

$$\mu_C = \mu + \mathrm{i}\omega\mu' \tag{1}$$

where ω is the angular frequency, μ is the modulus of rigidity and μ' is the viscoelastic parameter, according to the notation used by Dey *et al.*[1]

The correspondence principle[2] implies that the complex viscoelastic velocity can be obtained from the elastic velocity by substituting the elastic shear modulus μ with the corresponding complex modulus μ_C. Then, if C_1 denotes the complex velocity and k the complex wave number, we have that

$$C_1 = \frac{\omega}{k} = \sqrt{\frac{\mu_C}{\rho}} \tag{2}$$

Since $C_2 = (\mu/\rho)^{1/2}$ is the elastic velocity, equation (2) becomes

$$\frac{C_1}{C_2} = \sqrt{1 + \mathrm{i}\omega\frac{\mu'}{\mu}} = \sqrt{1 + \mathrm{i}A} \tag{3}$$

Using properties of complex numbers, it can be easily shown that equation (3) is identical to the dispersion equation (13) obtained by Dey *et al.*[1] (see also equation (5-99) in Ewing *et al.*[3]), indicating that they simply obtained the period equation of the torsional body wave propagating in an unbounded and homogeneous viscoelastic medium.

The condition by which the displacement $v \to 0$ for $z \to \infty$, probably interpreted as a necessary condition for the existence of a surface wave, is just indicating that the shear body wave is attenuated at very long depths. Equation (4) in Dey *et al.*[1] is the body wave displacement and equation (11) is the corresponding displacement for a twist of magnitude P applied on a circle of radius a on the surface of a viscoelastic half-space.

It is not clear what the authors mean by damped velocity and with the conclusion that the velocity of the torsional wave in a viscoelastic soil is less than that of the shear wave velocity in an elastic soil. Since the phase velocity V_p is equal to the frequency divided by the real part of the complex wave number k, the proper definition for the normalized phase velocity is

$$\frac{V_\mathrm{p}}{C_2} = \left[\mathrm{Re}\left(\frac{C_2}{C_1}\right)\right]^{-1} \tag{4}$$

Using equation (2) shows that the viscoelastic phase velocity is always greater than the elastic or relaxed ($\omega \to 0$) phase velocity C_2 (see also Reference 3). It is clear that if such viscoelastic surface

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torsional wave should exist, its phase velocity cannot be the elastic phase velocity in the absence of loss, as the authors claim in the last paragraph of page 211.

Effective ways for generating SH surface solitons were reviewed by Maugin.[4] In general, it is viewed as a perturbation in the boundary conditions, in order to bind otherwise essentially bulk SH waves to the limiting surface. These perturbations include spatial inhomogeneities, additional interfaces (Love waves), curvature, gratings, roughness, spatial dispersion, and couplings with electric and magnetic fields.

The use of viscoelastic rheology in a homogeneous half-space may cause a strong attenuation with depth but also along the radial distance, failing to produce the propagation of effective surface waves.

REFERENCES

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REPLY TO LETTER TO THE EDITOR

In this paper normalized velocity has been obtained in equation (13), (p. 211) Prof. Jose'M Carcione also agrees (vide his equation (13). The real part $\{(1 + A^2)^{1/2}/2 + 1/2\}^{1/2}$ shows the velocity at which the Torsional surface wave would have been propagated in the absence of damping and the imaginary part gives the actual damping of the wave. In case the real part of normalized velocity is more than the damping velocity, the wave will propagate. It is obvious that the real part of the normalized velocity is more than that of the imaginary part, showing that the torsional surface wave will propagate in the medium. The second point raised by the honourable analyser of the paper can be explained in the following way.

The actual velocity of propagation of Torsional surface wave can be calculated from Figure 1 taking the difference of real value of C_1/C_2 and imaginary value (C_1/C_2) at any frequency of propagation. From the curves of Figure 1 it is clear that $\mathrm{Re}(C_1/C_2) - \mathrm{Im}\,(C_1/C_2)$ is always less than unity and may become unity at low frequency. This shows that Actual velocity of propagation$/C_2 < 1$, meaning thereby that the velocity of propagation of Torsional surface wave in a viscoelastic medium is less than the velocity of shear wave in a corresponding elastic solid. Now as $\mu^1 \to 0$, Figure 1 suggests that damping will be less and less and ultimately $C_1 \to C_2$. The same may be obtained as well from equation (13) and, hence, for an elastic solid, the Torsional mode will coincide with the shear mode.

With the above analysis, I do not find any contradiction in the result. I must thank Prof. Jose'M Carcione for taking much interest on this paper and his constructive analysis.

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